Uncertainty-Based Semantics for Multi-Agent Knowing How Logics

Carlos Areces[a], Raul Fervari[a], Andrés R. Saravia[a] and Fernando R. Velázquez-Quesada[b]

[a]CONICET and Universidad Nacional de Córdoba, Argentina
[b]ILLC, Universiteit van Amsterdam, The Netherlands

Overview of the talk

- Knowing how logics: Some background
- Wang's original proposal: Knowing how over LTSs
- Our proposal: Knowing how over LTS^U
- Comparing the two semantics
- Our results: Axiomatization and complexity
- Conclusion and future work

Knowing How Logics: Some History

- **Epistemic Logic:** reasoning about knowledge of agents.
- E.g. *John knows that it is sunny in Paris*.
- Other patterns of knowledge: knowing why, knowing whether, knowing who and knowing how.
- Wang [2015,2018] proposed a framework for knowing how logics.

In our work:

- We generalize Wang's framework:
  - ***(1)*** Re-introduce the notion of epistemic indistinguishability.
  - ***(2)*** Enable multi-agent scenarios.
  - ***(3)*** Obtain a weaker, more general logic.

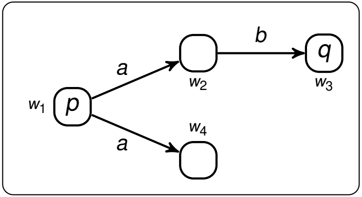
Labeled transition systems (LTSs)

Take Prop $\neq \emptyset$ a set of *propositions*, and Act $\neq \emptyset$ a the set of *actions*.

Definition (Labeled transition system)

$\mathcal{S} = \langle W, \{R_a\}_{a \in \mathrm{Act}}, V \rangle$ where

- $W \neq \emptyset$,
- $R_a \subseteq W \times W$,
- $V : W \to 2^{\mathrm{Prop}}$.



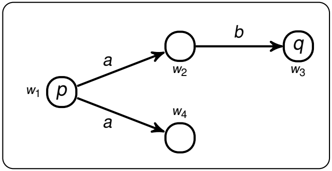
(from [Wang 2015,2018])

Strong executability

A plan should be fail proof:

Every partial execution should be completed.

Example:



ab is not strongly executable at w_1

Definition (Strong executability of a plan)

$\sigma \in \text{Act}^*$ is *strongly executable* (SE) at $u \in \text{W}$ iff, for all $k \in [0\,..\,|\sigma|-1]$,

$$v \in \text{R}_{\sigma_k}(u) \quad \text{implies} \quad \text{R}_{\sigma[k+1]}(v) \neq \emptyset.$$

Define $\text{SE}(\sigma) := \{w \in \text{W} \mid \sigma \text{ is SE at } w\}$.

L_{Kh} over LTS

Definition (Language L_{Kh})

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathrm{Kh}(\varphi,\varphi).$$

$\mathrm{Kh}(\psi,\varphi)$: *"when ψ holds, the agent knows how to make φ true"*.

Definition (L_{Kh} over LTS)

$\mathcal{S}, w \models \mathrm{Kh}(\psi,\varphi)$ *iff*$_{def}$ $\exists\sigma \in \mathrm{Act}^*$ such that

- **(Kh-1)** $[\![\psi]\!]^{\mathcal{S}} \subseteq \mathrm{SE}(\sigma)$ — *"σ is SE at all ψ-worlds"*
- **(Kh-2)** $\mathrm{R}_\sigma([\![\psi]\!]^{\mathcal{S}}) \subseteq [\![\varphi]\!]^{\mathcal{S}}$ — *"σ always ends in φ-worlds"*

where $[\![\varphi]\!]^{\mathcal{S}} := \{w \in W \mid \mathcal{S}, w \models \varphi\}$.

Notice that Kh is a global modality.

Axiom system $\mathcal{L}^{\text{LTS}}_{\text{Kh}}$: $\mathcal{L}$ + $\mathcal{L}_{\text{LTS}}$

$\mathcal{L}$:	TAUT	$\vdash \varphi$ being φ a propositional tautology
	MP	From $\vdash \varphi$ and $\vdash \varphi \rightarrow \psi$ infer $\vdash \psi$
	DISTA	$\vdash \mathsf{A}(\varphi \rightarrow \psi) \rightarrow (\mathsf{A}\varphi \rightarrow \mathsf{A}\psi)$
	NECA	From $\vdash \varphi$ infer $\vdash \mathsf{A}\varphi$
	TA	$\vdash \mathsf{A}\varphi \rightarrow \varphi$
	4KhA	$\vdash \mathsf{Kh}(\psi, \varphi) \rightarrow \mathsf{AKh}(\psi, \varphi)$
	5KhA	$\vdash \neg\mathsf{Kh}(\psi, \varphi) \rightarrow \mathsf{A}\neg\mathsf{Kh}(\psi, \varphi)$
$\mathcal{L}_{\text{LTS}}$:	EMP	$\vdash \mathsf{A}(\psi \rightarrow \varphi) \rightarrow \mathsf{Kh}(\psi, \varphi)$
	COMPKh	$\vdash (\mathsf{Kh}(\psi, \varphi) \wedge \mathsf{Kh}(\varphi, \chi)) \rightarrow \mathsf{Kh}(\psi, \chi)$

where $\mathsf{A}\varphi := \mathsf{Kh}(\neg\varphi, \bot)$, given that $\mathcal{S}, w \models \mathsf{Kh}(\neg\varphi, \bot)$ iff $[\![\varphi]\!]^{\mathcal{S}} = \mathsf{W}$.

The EMP axiom

Is $\models \mathsf{A}(\psi \to \varphi) \to \mathsf{Kh}(\psi, \varphi)$ adequate?

- Is a global implication enough to guarantee that the agent knows how to do something? In standard EL, φ does not imply $\mathsf{K}_i\varphi$.
- What if the needed plan (even *do nothing*) is not available?
- **The agent might be unaware of the existence of some plans.**

Many different reasons for not knowing how. The agent cannot distinguish between basic actions, the order, etc.

Uncertainty-based LTS (LTS^U)

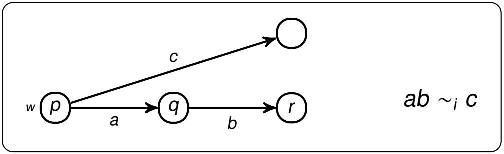
Take a finite non-empty set Agt.

Definition (Uncertainty-based LTS)

$\mathcal{M} = \langle W, \{R_a\}_{a \in Act}, \{\sim_i\}_{i \in Agt}, V \rangle$ where

- $\langle W, \{R_a\}_{a \in Act}, V \rangle$ is an LTS,
- $\sim_i$ is an equivalence relation over a non-empty $P_i \subseteq Act^*$.

For simplicity, define $[\sigma]_i := \{\sigma' \in P_i \mid \sigma \sim_i \sigma'\}$ and $S_i := \{[\sigma]_i \mid \sigma \in P_i\}$.



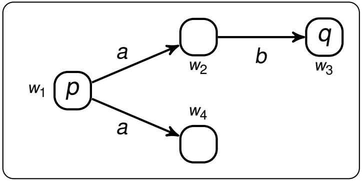
Strong executability *of a set of plans*

Definition (Strong executability of a set of plans)

$\pi \subseteq \text{Act}^*$ is *strongly executable* (SE) at $u \in W$ iff, for *all* $\sigma \in \pi$,

$$\sigma \text{ is } \textit{strongly executable} \text{ at } u$$

Define $\text{SE}(\pi) := \bigcap_{\sigma \in \pi} \text{SE}(\sigma)$.



- $w_1 \in \text{SE}(a)$
- $w_1 \notin \text{SE}(ab)$

$\Rightarrow\ w_1 \notin \text{SE}(\{a, ab\})$

$\mathsf{L_{Kh}}$ over LTS^{U}

Definition ($\mathsf{L_{Kh_i}}$ over LTS^{U})

$$\mathcal{M}, w \models \mathsf{Kh}_i(\psi, \varphi) \quad \textit{iff}_{def} \quad \exists \pi \in \mathsf{S}_i \text{ such that}$$

$$\textbf{(Kh-1)}\ [\![\psi]\!]^{\mathcal{M}} \subseteq \mathsf{SE}(\pi) \ \textit{and}$$

$$\textbf{(Kh-2)}\ \mathsf{R}_\pi([\![\psi]\!]^{\mathcal{M}}) \subseteq [\![\varphi]\!]^{\mathcal{M}}$$

with $[\![\varphi]\!]^{\mathcal{M}} := \{w \in \mathsf{W} \mid \mathcal{M}, w \models \varphi\}$.

Axiom system $\mathcal{L}^{\text{LTS}^\text{U}}_{\text{Kh}_i}$: $\mathcal{L} + \mathcal{L}_{\text{LTS}^\text{U}}$

$\mathcal{L}$:	TAUT	$\vdash \varphi$ being φ a propositional tautology
	MP	From $\vdash \varphi$ and $\vdash \varphi \to \psi$ infer $\vdash \psi$
	DISTA	$\vdash \text{A}(\varphi \to \psi) \to (\text{A}\varphi \to \text{A}\psi)$
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	TA	$\vdash \text{A}\varphi \to \varphi$
	4KhA	$\vdash \text{Kh}_i(\psi, \varphi) \to \text{AKh}_i(\psi, \varphi)$
	5KhA	$\vdash \neg\text{Kh}_i(\psi, \varphi) \to \text{A}\neg\text{Kh}_i(\psi, \varphi)$
$\mathcal{L}_{\text{LTS}^\text{U}}$:	KhE	$\vdash (\text{E}\psi \wedge \text{Kh}_i(\psi, \varphi)) \to \text{E}\varphi$
	KhA	$\vdash (\text{A}(\chi \to \psi) \wedge \text{Kh}_i(\psi, \varphi) \wedge \text{A}(\varphi \to \theta)) \to \text{Kh}_i(\chi, \theta)$

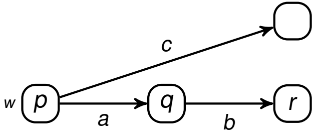
where $\text{A}\varphi := \bigvee_{i \in \text{Agt}} \text{Kh}_i(\neg\varphi, \bot)$

(given that $\exists i \in \text{Agt}$ with $\mathcal{M}, w \models \text{Kh}_i(\neg\varphi, \bot)$) iff $[\![\varphi]\!]^{\mathcal{M}} = \text{D}_{\mathcal{M}}$).

Comparing LTS and LTS^U

$\mathcal{L}_{\text{LTS}}$:	EMP	$\vdash \mathsf{A}(\psi \to \varphi) \to \mathsf{Kh}(\psi, \varphi)$
	COMPKh	$\vdash (\mathsf{Kh}(\psi, \varphi) \land \mathsf{Kh}(\varphi, \chi)) \to \mathsf{Kh}(\psi, \chi)$
$\mathcal{L}_{\text{LTS}^{\text{U}}}$:	KhE	$\vdash (\mathsf{E}\psi \land \mathsf{Kh}_i(\psi, \varphi)) \to \mathsf{E}\varphi$
	KhA	$\vdash (\mathsf{A}(\chi \to \psi) \land \mathsf{Kh}_i(\psi, \varphi) \land \mathsf{A}(\varphi \to \theta)) \to \mathsf{Kh}_i(\chi, \theta)$

- $\models_{\text{LTS}}$ KhE and $\models_{\text{LTS}}$ KhA.
- $\not\models_{\text{LTS}^{\text{U}}}$ EMP and $\not\models_{\text{LTS}^{\text{U}}}$ COMPKh:



$$S_i = \left\{ \begin{array}{c} \{a\},\ \{b\} \\ \{ab, c\} \end{array} \right\}$$

Can be recapture the LTS semantics with LTS^{U}?

Proposition

Given $\mathcal{S} = \langle \mathsf{W}, \{\mathsf{R}_a\}_{a \in \mathsf{Act}}, \mathsf{V} \rangle$, *define*

$$\mathcal{M}_{\mathcal{S}} = \langle \mathsf{W}, \{\mathsf{R}_a\}_{a \in \mathsf{Act}}, \{\{\sigma\} \mid \sigma \in \mathsf{Act}^*\}, \mathsf{V} \rangle.$$

Then, $[\![\varphi]\!]^{\mathcal{S}} = [\![\varphi]\!]^{\mathcal{M}_{\mathcal{S}}}$ *for every* $\varphi \in \mathsf{L}_{\mathsf{Kh}}$.

Proposition

Let $C := \{\mathcal{M}_{\mathcal{S}} \mid \mathcal{S}$ *is an LTS*$\}$. *Then,* $\mathcal{L}^{\mathsf{LTS}}_{\mathsf{Kh}}$ *is sound and strongly complete for* L_{Kh} *w.r.t. the class* C.

Complexity

Proposition

Let $\mathcal{M}^{\Gamma}, w$ be a canonical model and φ an $\mathsf{L}_{\mathsf{Kh}_i}$-formula. There is a submodel $\mathcal{M}'$ polynomial on the size of φ s.t. if $\mathcal{M}^{\Gamma}, w \models \varphi$ then $\mathcal{M}', w \models \varphi$.

Theorem

- *The model checking problem for $\mathsf{L}_{\mathsf{Kh}_i}$ is in* P.
- *The satisfiability problem for $\mathsf{L}_{\mathsf{Kh}_i}$ is* NP*-complete.*

Summary

- Uncertainty-based semantics for knowing how:
  - Indistinguishability among plans, for multiple agents.
  - Weaker, but more general logic.
  - Other reasons for **not** knowing how.
- Axiom system:
  - $\models_{\mathsf{LTS}^{\mathsf{U}}} (\mathsf{E}\psi \land \mathsf{Kh}_i(\psi,\varphi)) \to \mathsf{E}\varphi$,
  - $\models_{\mathsf{LTS}^{\mathsf{U}}} (\mathsf{A}(\chi \to \psi) \land \mathsf{Kh}_i(\psi,\varphi) \land \mathsf{A}(\varphi \to \theta)) \to \mathsf{Kh}_i(\chi,\theta)$.
- Complexity:
  - Model checking is in P.
  - Satisfiability checking is NP-complete.

Future work

- Combining knowing how + knowing that modalities,
- Dynamic modalities for learning/forgetting how.
- Notions of distributed knowledge.
- Exploiting the flexibility of our framework:
  - Other classes of models.
  - Different conditions of executability.
  - Other axioms.